# Lesson studies in initial mathematics teacher education

João Pedro da Ponte
Instituto de Educação, Universidade de Lisboa, Lisboa, Portugal

## Abstract

**Purpose** – The purpose of this paper is to review the research concerning the use of lesson studies in the education of secondary school prospective mathematics teachers. It discusses the adaptations made on the designs, the aims and outcomes established, the processes used, and the needed improvements in the experiences reported so far.
**Design/methodology/approach** – The scientific studies reviewed were identified in a Google search, using the key words "lesson study", "mathematics", and "secondary." The identified material was recorded in a database and the themes for the analysis cover the planning, execution, and reflection phases of a formative process.
**Findings** – The paper identifies the several pending issues regarding the use of lesson studies in prospective teacher education such as defining the aims, establishing the relationships among participants, scaling, and adapting lesson studies for the particular purpose of educating future teachers.
**Research limitations/implications** – At a practical level, this review suggests that lesson studies in pre-service teacher education must have a clear formative aim. It also shows that many formats are possible and must be chosen according to the specific conditions. In addition, it suggests the need for research regarding the definition of the aims, the working relationships established among participants, the problem of scale, and the problem of adaptation or simplification.
**Originality/value** – The paper identifies the key issues in the design of lesson studies in initial teacher education. It argues that besides signaling the positive outcomes, more critical (or self-critical) investigations are needed, e.g. using external researchers as "critical friends", which address their difficulties, limitations, and drawbacks in a more thorough way.

**Keywords**: Lesson study, Mathematics, Initial teacher education, Secondary
**Paper type:** Literature review

## 1. Introduction

In a lesson study, a group of teachers or a combined group of teachers and teacher educators/researchers work together, identifying students' difficulties on a given concept or issue, studying the related curriculum guidelines and research results, analyzing tasks, and planning what they consider as a suitable lesson to address the proposed concept or issue. This "research lesson" is taught by a member of the group to a class of students, whereas the others observe the lesson with a focus on student learning. The participants seek to verify to what extent this lesson achieves the sought objectives and what difficulties arise. Based on this analysis, they may revise the lesson and re-teach it to another class (Hart, Alston, and Murata, 2011). Lesson studies may involve several cycles of planning, teaching, observing, and revising (Robinson and Leikin, 2012), but the most important is that they are a practice-oriented and job-

embedded professional development approach. Given the focus and the nature of the process, lesson studies may be regarded as a small investigation of the participants carried out on their own professional practice.

Lesson study is a reflective and collaborative activity that combine practice and theory, promote a deep look of students' thinking as well as of curricular guidelines and research results on the relevant issue and this is important both for practicing and prospective teachers. However, given the nature of national policies of prospective teacher education, and the usual scarcity of time and resources available, their use in this process is rather problematic (Lamb and Ko, 2016). For all those involved in the preparation of secondary school prospective mathematics teachers and interested in exploring the potential of lesson studies the questions are relevant: are lesson studies suitable to introduce in their preparation? What can we learn from existing experiences? This paper reviews research studies concerning the use of lesson studies in the education of secondary school prospective mathematics teachers.

## 2. Method

The studies reviewed in this paper were identified in a Google search, using the key words "lesson study", "mathematics", and "secondary"[1]. The search included four kinds of publications: scientific journals, book chapters, congress proceedings, and PhD dissertations. From the material identified[2] in this way I selected and analyzed the 16 studies related to secondary school or combined secondary school and middle school prospective mathematics teacher education.[3]

In reviewing the papers, a data base was created with the entries: (i) Aims of the study; (ii) Framework, methodology and participants; (iii) Activities carried out as part of the lesson study, organization, and roles of the participants; and (iv) Conclusions and implications. The themes for the analysis emerged from this scheme of recording and are the following: (i) Adaptation of lesson study to initial teacher education; (ii) Aims

[1] The search that yielded results was carried out in English. Searches conducted in French, Spanish and Portuguese, with correspondent key words, yielded no results concerning prospective secondary school mathematics teachers. There were no time limits for the papers searched.

[2] The search identified 7 articles in scientific journals, 1 book chapter, 4 congress proceedings papers and 4 PhD dissertations.

[3] What is meant by "secondary school" changes from country to country. In some cases, there are "lower" and "upper" secondary school levels and in some other countries there is a combination of "middle" and "secondary" school. What is common in most cases is that the secondary school mathematics teacher is a specialist in the teaching of this subject.

and outcomes of lesson studies; (iii) Processes used in the lesson studies; and (iv) Needed improvements. It should be noted that points (i) and (ii) concern the design and planning of the lesson study, point (iii) concerns how it was carried out, and point (iv) what was the critical reflection (if any) from the researchers.

### *3. Features of the lesson studies*

Some important features of the 16 lesson studies reviewed in this paper are summarized in Table 1. Eight of these studies were carried out in the USA and the other eight in different countries. The table shows that the lesson studies reviewed have important distinctive differences. Two of them, Peterson, (2005) and Elipane (2012) are naturalistic studies carried out in Japan, whereas the remaining 14 are intervention studies in which the researchers created the lesson studies that are either the context or the object of study. Three of these intervention studies involve rather large groups of prospective teachers requiring special organizational arrangements. So, Radovic et al. (2014), with 50 participants, involved an aggregate of lesson studies carried out in 5 schools; Chew et al. (2014), with 46 participants, was conducted as a microteaching format[4]; and Burroughs and Luebeck (2010; with 24 participants was conducted in association with a group of five school teachers, who in fact carried out a lesson study whereas the participants acted mostly as observers. In addition to Chew et al. (2014), the microteaching format was also used in Fernandez (2005), with 18 participants, Fernandez and Zilliox (2011), with 36 participants in two years, and by Cavin (2006), with 6 participants. In two of the studies, Plummer and Peterson (2009) and Elipane (2012), only a single prospective teacher is the subject of study.

Insert Table 1

All these studies include some explanation about lesson studies (sometimes with considerable detail), but they vary a lot in terms of content and depth of theoretical frameworks (see Table 1), which may be used mostly to inform the conduction of the lesson study, to conduct the data analysis or to both. At an end of the spectrum, there

[4] Micro-teaching is a teacher education process in which a trainee acts as teacher of a small teaching unit (usually lasting less than 20 minutes), for a small group of pears, to practice some teaching technique or illustrate the teaching of some concept. An important aspect of microteaching is the constructive feedback obtained from peers and the discussion of the improvements that can be made to their teaching technique.

are some cases with no reference to theoretical framework al all, as in Peterson (2005), Burroughs and Luebeck (2010), and Gurl (2010). In an intermediate position, other studies include some reference to theoretical frameworks but they do not seem to play a very important role, such as Plummer and Peterson (2009) who refer to cultural beliefs about teaching and learning and Elipane (2012) who considers, among others, cognitive and sociocultural learning theories and Habermas' theory of human interests. In a stronger situation, some studies clearly draw on some theoretical notions, such as Hughes (2006), with the mathematical tasks framework, Mostofo (2013) with self-efficacy and Vygostky space, and Ponte et al. (2015) with levels of curriculum development. At the other end of the spectrum, some studies show a clear theoretical orientation, especially in conduction the lesson studies, notably Ricks (2011) and Radovic et al. (2014) with the notions of reflection and reflective practice and Cavin (2006) and Chew et al. (2014) with the notion of TPCK – Technological Pedagogical Content Knowledge. The notion of learning community was also very important in the framing of the studies Cavanagh and Garvey (2012), Fernandez and Zilliox (2011), and Gunnarsdóttir and Páldóttir (2011).

It is interesting to note that issues of mathematical content were not discussed at length in these studies. The topics considered varied a lot, with predominance of algebra concepts and procedures (Table 1). In several cases, such as the microteaching study of Fernandez (2005) and Fernandez and Zilliox (2011) there were several topics tackled in parallel by different groups of PT. It is also interesting to note that mathematical processes were at focus on four studies, Plumer and Peterson (2009), mathematical justification, Fernandez and Zilliox (2011), mathematical reasoning, Ricks (2011), mathematical communication, Ponte et al. (2015), mathematical reasoning and communication, and Cavanagh and Garvey (2012), problem solving.

## 4. Adaptation of lesson study to initial teacher education

Given the specific constrains of prospective teachers' education, lesson studies need to be suitably adapted. Prospective teachers are not fully certified teachers and it may be unwise that they teach regular lessons, at least in some stages of their preparation (that may even be unethical regarding children). Another issue is the difficulty in establishing truly collaborative relationships among participants, given their different status. In fact, teacher educators and school mentors/cooperating teachers are established professionals in their fields whereas prospective teachers are just getting prepared to begin their

professional career, and, therefore, their level of knowledge and experience is much different. In addition, teacher educators and school mentors have the power to make the key decisions regarding the teacher education program and, in particular, regarding the conduction of the lesson study process. They may withhold partially such power, but they will assume it at any stage if they think that is necessary. Such imbalance of experience and professional status among the participants does not exist, or at least does not exist in a similar way, in lesson studies with practicing teachers.

Lesson studies with prospective secondary school mathematics teachers may be carried out in different stages of the initial teacher education program. However, they are far more common just towards the end of the programs. It must be noted that at the earlier stages of the teacher education program prospective teachers are more likely to still miss some important skills and competencies to teach the research lesson, whereas at the very end teaching lessons is part of their usual preparation as teachers.

Carried out in initial teacher education, there are several possibilities for organization of lesson studies, concerning the role that prospective teachers assume and their relation with the schools' mentor teachers. In some cases, the prospective teachers have the main responsibility in planning and in conducting the lesson (Ricks, 2011; Cavanagh and Garvey, 2012; Gunnarsdóttir and Pálsdóttir, 2011; Mostofo, 2013) and the mentor teachers act mainly as consultants. In other cases, they have well-defined roles in planning and in reflecting about the lesson, but the lesson is taught by a school teacher (Burroughs and Luebeck, 2010; Ponte et al., 2015). And, still in other cases, the lesson study just involves prospective teachers planning lessons and teaching to each other and finally reflecting on these lessons, with no participation of school children (Fernández, 2005; Cavin, 2007; Fernández and Zilliox, 2011; Chew, Sama, Yew, and Lian, 2014).

## 5. Aims and outcomes of lesson studies

To be able to learn from lesson studies we need to realize what their aims are, that is, what is sought for the development of prospective teachers. However, such aims are not indicated in an explicit way in most studies and have to be inferred from the research design and results. Having this in mind, we note that some of the lesson studies are directed towards the development of aims situated essentially at a professional level, such as prospective teachers' knowledge about how to conduct teaching in alignment with curriculum reform orientations (Fernández, 2005; Fernández and Zilliox, 2011).

Ricks (2011) puts the emphasis in the development of prospective teachers' reflective ability through several reflective/investigative cycles, in which they also develop their ability to build lesson plans, showing progressive refinement of ideas. Another example is Cavanagh and Garvey (2012) that stress the increase in the ability of prospective teachers to make strong connections between theory and practice, learn from each other in a learning community, and reinforce their perspective about reform-oriented mathematics teaching based on a problem-solving approach. Still another example is the study of Radovic at al. (2014) that put the focus on prospective teachers' development of reflective practice. Taking into account his observations of a prospective teacher field work in a Japanese school, Elipane (2012) also indicates "skills, competencies, and habits of mind" that seemed to be addressed on that setting and that are essentially related to mathematics teaching issues: (i) getting acquainted with school contexts and classroom norms; (ii) learning to use in a meaning way powerful resources in the classroom; (iii) using the classroom and the school as sites for inquiry; (iv) getting involved in critical reflections; and (v) developing capacity for collaboration.

Other studies are targeted at specific didactical aspects. In this group an important aim in Hughes (2006) and Burroughs and Luebeck (2010) is learning how to plan lessons. In addition, Burroughs and Luebeck (2010) emphasize the importance of assessing students' prior knowledge in making such plans. Other studies, such as Cavin (2007) and Chew at al. (2014) address quite specific aims as developing particular issues of didactical knowledge such as TPACK for teaching mathematics with technology.

In addition, several studies indicate aims both at the professional and didactical level. For example, in the study of Gunnarsdóttir and Pálsdóttir (2011) the focus was in prospective teachers developing their professional language, using more professional concepts and with stronger reference to literature connecting theory and practice, and developing collaboration competence, seeing themselves as a part of learning community and assuming that they may learn a lot from others and by working with others. In the case of Mostofo (2013), an important specific goal was to improve prospective teachers' ability to make lesson plans but a more general goal was to support their professional growth, increasing their instructional effectiveness as well as their confidence in teaching. Still other example is the study of Ponte et al. (2015) who address the prospective teachers' learning of lesson planning (with emphasis on

mathematical tasks and classroom communication) as well on the development of the ability to reflect on the lessons.

Only in one study, conducted by Pummer and Petterson (2009), the learning or refining of mathematics content knowledge is the main research issue and seemed to be the main aim behind the lesson study. However, a significant result that several of the studies referred above indicate is that prospective teachers experienced significant learning of knowledge of mathematics content and processes (Fernández, 2005; Fernández and Zilliox, 2011; Ponte et al., 2015), the improved elaboration of the subject matter (Gunnarsdóttir and Pálsdóttir, 2011) and the expansion of the participants' views on mathematical knowledge (Cavin, 2007).

## 6. Processes used in lesson studies

Another way of looking at lesson studies in initial teacher education is to see where they put the focus in terms of the lesson study process. In some cases, this is put clearly in the planning stage (as in Hughes, 2006; Burroughs and Luebeck, 2010; Cavanagh and Garvey 2012; Gunnarsdóttir and Pálsdóttir, 2011; Ponte at al., 2015), in other cases this is put in the analysis and reformulation of the lesson (Fernández, 2005; Fernández and Zilliox, 2011; Mostofo, 2013; Ricks, 2011; Radovic et al., 2014), and still in other cases it seems that there is an attempt to balance both (Gurl, 2010; Mostofo, 2013). Accordingly, the main activities may be studying the mathematics, the mathematics curriculum orientations, and previous research on students' difficulties on the topic or discussing actual classroom events and their possible implications for the revision of the lesson.

An emphasis in the planning stage, for example, may be traced in the study of Cavanagh and Garvey (2012). However, this study had several features that make it quite distinctive from others. One of these features was the establishment of a learning community, including the nine prospective teachers, the university supervisor and the school mentor who both acted in the role of co-constructors of knowledge. This learning community lasted for the whole school year of the teacher education program and developed around visits to the school (six visits in each semester) as well as an online virtual group. Another feature of this case is that it is based on the mutual interest of the university researcher and the school mentor in establishing a project partnership. As a third important feature, is that the lessons were co-taught by pairs of prospective teachers, so that each prospective teacher co-taught two or three lessons. This co-

teaching by prospective teachers is rather unique of this experience and the authors considered that it was very fruitful. The prospective teachers were very positive about this experience, considering that this enhanced their creativity and confidence. There was a short oral reflection after each lesson and also an individual personal reflection to an online forum. The prospective teachers found this task more challenging but also more rewarding than the oral reflection in group and the authors claim that this personal reflection made prospective teachers to think more deeply about what they observed.

In the study of Gunnarsdóttir and Pálsdóttir (2011) the emphasis was also in the planning stage. However, there was also a distinctive feature in this study given that some prospective teachers were distance learners and together with the colleagues that were regular attendants of sessions they shared a digital learning environment. Records of work in progress from face-to-face classes regarding the planning of the research lesson, were shared in this way with distance learners that also contributed to their improvement. The first version of the lesson plans received comments from the school mentors.

Also in the study of Ponte et al. (2015) the emphasis was on the planning, which was undertaken in a quite thorough manner for five three hour sessions. The class of seven prospective teachers was divided in two groups which led to the joint planning of two lessons that were taught by the mentor teacher and observed by all participants. These lessons were then object of an oral reflection (in two three hour sessions). There was also a written reflection done by the two groups concerning the whole lesson study process.

An emphasis on reflection is found in the study of Ricks (2011), carried in the frame of a mathematics methods course. Prospective teachers worked in groups of four preparing a lesson to teach to their colleagues of the other groups. This lesson, after revision, was then taught to public high school students. The prospective teachers made a final group report describing the whole process, including the changes made in the lesson plan and further changes that they propose after the public school teaching. The main idea was to lead them through a reflection process that from an experiential event, evolved through phases of idea suspension and problem creation, idea formation, testing acting with observation, to a new cycle of idea suspension and problem creation, and so on.

Reflection is also the focus in the study of Radovic et al. (2014). This study involved 50 prospective secondary mathematics teachers and practicing teachers from

six schools with different organizational situations in different groups. In four cases the practicing teachers had little involvement in the planning and the research lesson was taught by the prospective teachers whereas in two cases the practicing teachers had a strong involvement in the planning and they taught the research lesson. The authors indicate that the prospective teachers appreciated better the last situation and conclude "that engaging the participants in equal terms, sharing their roles and experiences, helps in providing a collective experience of [lesson studies]". Concerning the activities of the lesson study, they argue "that this role of imagination in reflecting on practice (after the lesson) is crucial to the students [prospective teachers], and that the re-teaching in lesson study may play a vital developmental role in this" (p. 277).

Many studies seem to strive for a balance between planning and reflecting. The study reported by Gurl (2010) involves an explicit adaptation regarding what the author describes as the "formal" Japanese lesson study format. This adaptation essentially meant simplifications both in the planning and in the reflection stages of the process. In fact, the construction of the lesson plan was done in one case in a 75 minute meeting in the same day that the lesson was taught and in another case in two meetings, one lasting for 45 minutes taking place in the day before the lesson and the other lasting 25 minutes in the day of the lesson. The reflection on the lesson was also carried out less formally that in the Japanese style, being done immediately after the lesson or in one or two sessions shortly after the lesson. The author regards this as a successful adaptation to the USA context since it could work in real settings without extra resources but one wonders if this rather simplified version still keeps the major features and potential benefits of a lesson study.

Also a balance between planning and reflecting may be found in the study of Mostofo (2013). In this case the prospective teachers carried out several cycles of collaborative lesson planning, testing these lessons by teaching them to each other, revising the lessons, and, finally, teaching them to Algebra I students, with a whole class discussion. In this process the prospective teachers received feedback from their colleagues and instructor and, in some cases, the school collaborating teacher. A similar format was used in the study of Plummer and Peterson (2009). The prospective teachers planned a lesson guided by a public school teacher. They chose the goal for the lesson and met during two hours for two weeks to plan the lesson. After that one member of the group taught the lesson to their colleagues whereas others observed the classroom events. The lesson was further refined for six more weeks and then another prospective

teacher taught it to high school students. Finally, each group made a report on the development of the lesson and the school students' responses to the lesson.

A balance between planning and reflecting also underlines the studies that used a microteaching format such as those carried out by Fernández and Zilliox (2011; see also Fernández 2005), Cavin (2007) and Chew at al. (2014). In the Fernández and Zilliox (2011) study the lesson studies were carried by groups of six prospective teachers and the mathematics content for the research lesson was chosen by the instructor taking into account their views of the participants' weaknesses concerning mathematics content knowledge. For each group, there were three cycles of planning, teaching the lesson, reflection (using videotapes), and revision. There was collaboration within each group and consultations with the instructor. The authors indicate that making a reflective report on the "lesson study work demonstrating growth in understanding teaching for the overarching mathematical students' learning goals . . . Appeared to be essential with respect to the learner-centeredness of the lesson study approaches" (p. 99). The study of Cavin (2007) used also microteaching in a similar arrangement, but in this case the participants were prospective teachers enrolled in a college mathematics education class dealing with classroom use of technology.

A similar arrangement may be found in the study of Chew et al. (2014) that involved 46 prospective secondary teachers in a mathematics teaching methods course in a Malaysian public university. The prospective teachers worked in groups of two in a sequence of (i) planning a lesson in collaboration (including GSP[5] sketches and students' tasks); (ii) seeing the lesson in action taught by the partner; (iii) discussing the lesson plan with the partner and instructor; (iv) revising the lesson plan; (v) teaching the new version of the lesson and getting feedback; and (vi) revising the lesson plan.

When reflection is an important element of the process, it often is done in two ways, first orally in a meeting shortly after the research lesson, and on a second moment by writing, such as in the case of Fernández and Zilliox (2011), Cavanagh and Garvey (2012) and Ponte et al. (2015). Another aspect that strengthens the element of reflection is the existence of more than one cycle of planning, carrying out the lesson and reflecting about it. In some cases this is done first on a microteaching format, with other prospective teachers, and afterwards with actual school students (e.g., Mostofo, 2013).

[5] GSP – The Geometer's Sketchpad, is a dynamic geometry software widely used in mathematics education that allows for making geometrical constructions in order to solve geometrical problems and to study the properties of geometrical figures.

All the studies referred to so far are experimental studies involving a researcher or a research team that carried out a lesson study. Two other studies, conducted by Peterson (2005) and Elipane (2012) involve naturalistic studies in Japan about student teaching in the preparation of prospective teachers. These studies do not describe lesson studies carried out in initial teacher education, but show several elements of parts of the lesson study process that are present in prospective teacher preparation and prepare the prospective teacher to carry out his/her professional role as well as to enter the lesson study culture. Peterson (2005) observed for two weeks prospective teachers' interaction with their mentor teachers in three different universities and practicing schools. He underlines the importance of the preparation of lessons, teaching, and reflection meetings (*hanseikai*) in student teaching experiences. Elipane (2012) conducted a case study of a prospective teacher that he closely observed for four weeks in his activity in the cooperating school. In his work, he refers four mechanisms that supported changes in this prospective teacher: (i) becoming aware of images of recommended teaching ("sensitization to images of reform"); (ii) "forged reifications of learning experiences"; (iii) "student feedback and communications"; and (iv) "immersion in communities of practice" (p. 230).

**7. Needed Improvements**

None of the research studies reported significant problems or need for improvement. However, some of the studies indicated weaknesses that should be addressed in future experiences. The most self-critical is the study of Radovic et al. (2014), in which the prospective teachers indicated that "they found the collaborative planning difficult and not very effective" (p. 274). This contrasts with the post-lesson discussion that most of them found easier as a context for developing reflection about practice. In the cases where the practicing teachers had less involvement, the prospective teachers found also more difficult to reflect about actual lessons. It must be noted that there are not many details in this paper about how the planning was carried out (in some cases in seems that the plan was already preexistent). One wonders if the common four column plan used in lesson studies (e.g., Roback, Chance, Legler, and Moore, 2006) was developed taking into account all possible students' difficulties, as is the case in many lesson studies. The authors suggest that "more guidance in reflecting/imagining [alternatives to the lesson] from experienced teachers is needed" (p. 277).

Cavanagh and Garvey (2012) report that there was a strong positive evaluation of the prospective teachers' learning and also about the suitability of the program features. Even so, there were some comments from the prospective teachers (echoed by the authors) that the design of an instrument for observing classes would have be useful to focus observations in important issues and that written reflections "became 'rather repetitive'" (p. 68). Taking into account that the reflection after the lesson was very brief, this means that there is some room for improvement both in the observation and on the reflection stages of this process. Also, in the study of Mostofo (2013) no major points of needed improvement were identified but the researcher indicated points of concern that should be taken into account in this activity: the prospective teachers' class size (if too large), the working teams dynamics, the partnership with local schools, and the communication with the school cooperating teacher.

Using lesson studies in initial teacher education is not a simple issue. Many decisions have to be taken by the course instructors regarding the adaptation of lesson study format to the particular situation, involving trade-offs between potential benefits and losses regarding other activities. Therefore, more critical (or self-critical) investigations are also needed, perhaps using external researchers as "critical friends" that are not so committed to focusing just on the benefits of a particular lesson study process and are also able to notice their difficulties, limitations and drawbacks.

## 8. Conclusion

This review of studies about the use of lesson study in mathematics prospective teacher education found a wide variety of situations which suggest some practical implications as well as some issues for further research in order to know better the possible affordances and pitfalls of this teacher education format and the conditions that make it a fruitful learning experience for the participants.

One of the issues is the definition of the aims of the lesson studies. A single lesson study, carefully planned and conducted may achieve many objectives, but cannot achieve the whole spectrum of aims of prospective teacher preparation. The aims may involve both professional and didactical issues but these must be clearly identified. For example, didactical aims should focus on specific learning goals (such as promoting students' reasoning), designing tasks, selecting and using materials (including technology), questioning students, and so on. Looking at tasks and mathematical content suggests an emphasis on planning, looking at classroom communication and

classroom management points towards a strong emphasis on reflection and re-teaching. Still on the issue of aims, we must note that very few studies addressed prospective teachers' development in relation to the mathematical content. This raises the question if lesson studies are not fitted to such purpose or such is a possibility that has been undervalued by researchers. Radovic at al. (2014) indicate that future research must deal with the question of "how best facilitate prospective teachers' experiences in their 'zone of professional development'" (p. 277) but one needs to know what professional development aims are on target for each particular group of prospective teachers.

A second issue refers to the working relationships that are established among participants. Prospective teachers are not equal to experienced teachers nor to teacher educators and researchers. In consequence, they need some guidance and perhaps more structure than regular participants in lessons studies with practicing teachers. However, to grow and to develop professionally, assuming responsibility for their actions, prospective teachers need to have some freedom to make decisions and assume risks and need to see that their ideas are valued and taken into account. These working relationships among participants are highly dependent on the educational culture and, in a more specific way, on the direction provided by the teacher educator that leads the process and the level of collaboration that is established among participants. Some studies highlighted the notion of learning community (Cavanagh and Garvey, 2012; Gunnarsdóttir and Pálsdóttir, 2011) whereas others emphasized structure (Burroughs and Luebeck, 2010; Fernández and Zilliox, 2011; Ricks, 2011). Therefore, the problem of establishing and adjusting working relationships in the lesson study process is quite critical and deserves more attention in future studies.

A third issue is the problem of scale. Dealing with a high number of prospective teachers creates many logistical problems since lesson study is an intensive collaborative activity. A high number of prospective teachers requires not only a high number of teacher educators (or a few teacher educators with plenty of time…) ready to supervise the whole project but also a high number of mentor teachers willing to participate and ready to commit themselves. The study of Radovic et al. (2014) seemed to tackle this issue by creating some flexibility in the arrangements but the authors showed concern with what may be lost by doing this. The studies that take a microteaching approach, such those of Fernandez (2005) and Chew et al. (2014) provide another solution to this issue, providing teacher educators with a strong control of the whole process by the establishment of clear rules about what to do in each step.

In this case, the danger lies in losing the connection with real schools and real students and also in making this process looking artificial to participants.

Still a fourth issue is the problem of adaptation or simplification. As the standard lesson study format is quite time consuming – especially in the planning stage and also in the repeated cycles of improving and re-teaching lessons – there is high pressure to simplify the process. In the study of Gurl (2010) the simplification amounted to just 70-75 minutes of preparation time (in one or two sessions) and also to a rather informal style of reflection. In the study of Hughes (2006) the simplification was just to carry out the planning with no actual teaching of the lesson. In the study of Ponte at al. (2015) there was extensive planning, teaching and observing the research lesson, as well as reflection, but there was no actual redesign of the tasks and trial in a second cycle. In Japan, it is common to make a strong simplification of the lesson study process in the education of prospective teachers, breaking it into small pieces (Elipane, 2012; Peterson, 2005). Without some simplifications the lesson study may not fit in the teacher education program, making impossible to achieve other important aims of the preparation of prospective teachers. However, too much simplification in the planning or in the reflection stage may weaken seriously the benefits of the process. The studies reviewed in this paper provide suggestions about possible simplifications and adaptations without losing too much of the benefits of lesson study but further research on this issue is in order.

In summary, lesson studies may be a very appropriate format for initial teacher education programs, providing prospective teachers with an opportunity to carry out activities that enable connecting theory and practice, researching in the professional practice, and, in addition, to review and develop their knowledge of mathematics concepts and processes. Their suitability for each context poses interesting problems for teacher educators, and the learning that prospective teachers may experience provides room for much further research projects.

## References

Burroughs, E. A. and Luebeck, J. L. (2010), Pre-service teachers in mathematics lesson study. *The Montana Mathematics Enthusiast*, Vol. 7, No. 2-3, pp. 391-400.

Cavanagh, M. S. and Garvey, T. (2012), A professional experience learning community for pre-service secondary mathematics teachers. *Australian Journal of Teacher Education*, Vol. 37, No. 12, pp. 57-75.

Cavin, R. (2006), *Developing technological pedagogical content knowledge in preservice teachers through microteaching lesson study* (Unpublished doctoral dissertation, Florida State University, USA), avaliable at http://diginole.lib.fsu.edu/cgi/viewcontent.cgi?article=2024andcontext=etd (assessed at 14 august 2016).

Chew, C. M., Lim, C. S., Wun, T. Y. and Lim, H. L. (2014), Effect of lesson on study on pre-service secondary teachers' technological pedagogical content knowledge *Sains Humanika*, Vol. 2, No. 4, pp. 55-61.

Elipane, L. (2012), *Integrating the essential elements of lesson study in preservice mathematics teacher education* (Unpublished doctoral dissertation, University of Copenhagen, Denmark), available at http://goo.gl/LWvwX5 (assessed at 01 august 2015).

Fernández, M. L. (2005), Exploring "lesson study" in teacher preparation. In H. L. Chick and J. L. Vincent (Eds.), *Proceedings of the 29th PME International Conference* (Vol. 2, pp. 305-310). Melbourne.

Fernández, M. L. and Zilliox, J. (2011), "Investigating approaches to lesson study in prospective mathematics teacher education". In Hart, L. C., Alston, A. and Murata, A. (Eds.), *Lesson study, research and practice in mathematics education*, Springer, Dordrecht, pp. 85-102.

Gunnarsdóttir, G. H. and Pálsdóttir, G. (2011), Lesson study in teacher education: A tool to establish a learning community. In Pytlak, M., Rowland, T., and Swoboda, E. (Eds.), *Proceedings of CERME 7*, pp. 2660-2669. University of Rzeszów, Poland.

Gurl, T. (2010), Improving preservice field placements in secondary mathematics: A residency model for student teaching through lesson study. *Journal of Mathematics Education at Teachers College*, Vol. 1, pp. 17-20.

Hart, L. C., Alston, A. and Murata, A. (Eds.) (2011), *Lesson study, research and practice in mathematics education*. Springer, Dordrecht.

Hughes, E. K. (2006), *Lesson planning as a vehicle for developing pre-service secondary teachers' capacity to focus on students' mathematical thinking* (Doctoral dissertation, University of Pittsburgh, USA), available at http://core.ac.uk/download/pdf/12210018.pdf (assessed 14 august 2016).

Lamb, P. and Ko, P. Y. (2016), Case studies of lesson and learning study in initial teacher education programmes. *International Journal for Lesson and Learning Studies*, Vol. 5, No. 2, pp. 78-83.

Mostofo, J. (2013), *Using lesson study with preservice secondary mathematics teachers: Effects on instruction, planning, and efficacy to teach mathematics* (Doctoral dissertation, Arizona State University, USA), available at http://repository.asu.edu/items/16460 (assessed 14 august 2016).

Peterson, B. (2005), Student teaching in Japan: The lesson. *Journal of Mathematics Teacher Education*, Vol. 8, No 1, pp. 61-74.

Plummer, J. S. and Peterson, B. E. (2009), A preservice secondary teacher's moves to protect her view of herself as a mathematics expert. *School Science and Mathematics*, Vol. *109*, No. 5, pp. 247-258.

Ponte, J. P., Quaresma, M., Mata-Pereira, J. and Baptista, M. (2015), Lesson study and curriculum development. Paper presented at the *II Conference on Curriculum Studies*. Porto, Portugal, available at http://www.fpce.up.pt/eccs2015/tables/CurriculumStudies_E-Book.pdf (assessed 14 august 2016).

Radovic, D., Archer, R., Leask, D., Morgan, S., Pope, S. and Williams, J. (2014), Lesson study as a Zone of Professional Development in secondary mathematics ITE: From reflection to reflection-and-imagination. In Pope, S. (Ed.), *Proceedings of the 8th British Congress of Mathematics Education*, pp. 271-278, available at http://www.bsrlm.org.uk/IPs/ip36-1/BSRLM-CP-36-1-03.pdf (assessed 14 august 2016).

Ricks, T. E. (2011), Process reflection during Japanese lesson study experiences by prospective secondary mathematics teachers. *Journal of Mathematics Teacher Education,* Vol. 14, No. 4, pp. 251-267.

Roback, P., Chance, B., Legler, J. and Moore, T. (2006), Applying Japanese lesson study principles to an upper-level undergraduate statistics course. *Journal of Statistics Education,* Vol. 14, No 2, pp. 1-23.

Robinson, N. and Leikin, R. (2012), One teacher, two lessons: The lesson study process. *International Journal of Science and Mathematics Education,* Vol. 10, No. 1, pp. 139-161.

Table 1

Features of the reviewed studies

| | | Country, Participant prospective teachers (PT) | Context and design | Theoretical framework (other than "lesson study") | Mathematical content addressed in the lesson study |
|---|---|---|---|---|---|
| Burroughs, E. A. and Luebeck, J. L. (2010), Pre-service teachers in mathematics lesson study. | *The Montana Mathematics Enthusiast.* | Ireland<br>16 elementary PT<br>8 secondary PT<br>5 school teachers<br>2 Researchers/TE | Mixed group of prospective and in-service teachers. | Not explicit. | Slope (in the context of parallel and perpendicular lines) (grade 8). |
| Cavanagh, M. S. and Garvey, T. (2012), A professional experience learning community for pre-service secondary mathematics teachers. | *Australian Journal of Teacher Education.* | Australia<br>9 secondary PT<br>1 school teacher<br>1 researcher/TE | Context of a learning community partnership between a school and a university. | Reflective professional experience (Le Cornu and Ewing, 2008).<br>Learning community/community of practice (Lave & Wenger, 1991; Wenger, 1998). | Problem solving (grade 8). |
| Cavin, R. (2006), *Developing technological pedagogical content knowledge in preservice teachers through microteaching lesson study* | *Doctoral dissertation, Florida State University.* | USA<br>5 math ed majors<br>1 science ed major | Microteaching LS. | Technological Pedagogical Content Knowledge (TPCK) (Mishra & Koehler, 2006). | Exploring mathematical patterns and/or relationships using technological tools. |
| Chew, C. M., Lim, C. S., Wun, T. Y. and Lim, H. L. (2014), Effect of lesson on study on pre-service secondary teachers' technological pedagogical content knowledge. | *Sains Humanika.* | Malasia<br>46 secondary PT | Microteaching LS in 6 steps (23 groups). | Technological Pedagogical Content Knowledge (TPCK) (Mishra & Koehler, 2006). | Each LS group was allowed to choose a topic in Algebra, Geometry, Statistics or Trigonometry. |
| Elipane, L. (2012), *Integrating the essential elements of lesson study in preservice mathematics teacher education*. | *Doctoral dissertation, University of Copenhagen.* | Japan<br>1 PT (main study) | Naturalistic study focused on a PT (4 weeks of observation). | Combining cognitive and sociological perspectives (Cobb, 2000).<br>Theory of human interests (Habermas, 1972). | Subtraction of positive and negative numbers. |
| Fernández, M. L. (2005), Exploring "lesson study" in teacher preparation. | *Proceedings 29th PME International Conference* | USA<br>18 secondary PT | Microteaching LS (6 groups). | Cognition (Resnick, 1987). | Fractals, traceable paths, Euler's formula, permutations, prisms and pyramids, and ellipses. |
| Fernández, M. L. and Zilliox, J. (2011), "Investigating approaches to lesson study in prospective mathematics teacher education". | In Hart, Alston, and Murata. (Eds.), *Lesson study, research and practice in mathematics education* | USA<br>36 PT | Microteaching LS (12 groups). | Sociocultutral theory (Vygostsky, 1978).<br>Learning community (Frykholm, 1998).<br>Effective learning environments (Bransford et al. 2000). | Mathematical reasoning. Ability to study patterns in constructing and justifying relationships or concepts. |

| | | | | | |
|---|---|---|---|---|---|
| Gunnarsdóttir, G. H. and Pálsdóttir, G. (2011), Lesson study in teacher education: A tool to establish a learning community. | In *Proceedings CERME 7*. | Iceland<br>30 PT | Some of the PT are distance learners and all PT share a digital learning environment. | Professional learning communities (Hammerness, Darling-Hammond, & Bransford, 2005). | Prime numbers. |
| Gurl, T. (2010), Improving preservice field placements in secondary mathematics: A residency model for student teaching through lesson study. | *Journal of Mathematics Education at Teachers College.* | USA<br>6 PT (3 pairs) | "Compacted LS". | Not explicit. | NA. |
| Hughes, E. K. (2006), *Lesson planning as a vehicle for developing pre-service secondary teachers' capacity to focus on students' mathematical thinking.* | *Doctoral dissertation, University of Pittsburgh* | USA<br>10 PT | Focus on planning. | The mathematical tasks framework (Stein, Grover, & Henningsen, 1996). Attention to students' mathematical thinking (Stigler & Hiebert, 1999). | Quadratic functions. |
| Mostofo, J. (2013), *Using lesson study with preservice secondary mathematics teachers: Effects on instruction, planning, and efficacy to teach mathematics.* | *Doctoral dissertation, Arizona State University.* | USA<br>6 PT | Action research study using qualitative and quantitative methods. | Self-efficacy (Bandura, 1993). Vygotsky Space (Gallucci, DeVoogt, Van Lare, Yoon, & Boatright, 2010). | Selected Algebra I topics. |
| Peterson, B. (2005), Student teaching in Japan: The lesson. | *Journal of Mathematics Teacher Education*. | Japan<br>Number of PT not indicated.<br>3 schools visited | Naturalistic study based in a 2.week visit to Japan. | Not explicit. | Systems of linear equations. |
| Plummer, J. S. and Peterson, B. E. (2009), A preservice secondary teacher's moves to protect her view of herself as a mathematics expert. | *School Science and Mathematics.* | USA<br>1 PT | Case study of a PT. | Cultural beliefs about teaching and learning mathematics (Stigler & Hiebert, 1999). | Logarithms.<br>Mathematical justification. |
| Ponte, J. P., Quaresma, M., Mata-Pereira, J. and Baptista, M. (2015), Lesson study and curriculum development. | *II Conference on Curriculum Studies*. | Portugal<br>7 PT | Two groups pf PT working with a single cooperating teacher. | Levels of curriculum development and implementations (Husen, 1967). | Similarity of triangles. Mathematical reasoning and communication. |
| Radovic, D., Archer, R., Leask, D., Morgan, S., Pope, S. and Williams, J. (2014), Lesson study as a Zone of Professional Development in secondary mathematics ITE: From reflection to reflection-and-imagination. | *Proceedings 8th British Congress of Mathematics Education*. | UK<br>50 PT | Large scale LS. | Reflective practice (Dewey, 1933; Schön, 1987). | Several topics at five participating schools. |
| Ricks, T. E. (2011), Process reflection during Japanese lesson study experiences by prospective secondary mathematics teachers. | *Journal of Mathematics Teacher Education.* | USA<br>4 PT | LS as a context to study "process reflection". | Reflection (Dewey, 1933; Schön, 1983) | Trigonometry (angles greater than 90 degrees). Mathematical communication (student sharing). |